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Problem sheet 4

Due date: Nov. 10, 2017.

Problem 13

Show that $K = \widehat{\mathbb{Q}_p(\mu_{p^\infty})}$ is perfectoid with $K^\flat = \left(\varprojlim_n \mathbb{F}_p[t^{\frac{1}{p^\infty}}]/(t^n)\right)[t^{-1}]$.

Problem 14

1. For every finite field extension K of $\mathbb{Q}_p$, the ring $(K^o)^\flat$ is the residue field.
2. Let $K = \widehat{\mathbb{Q}_p^{\mathrm{ur}}}$ be the completion of the maximal unramified extension of $\mathbb{Q}_p$. Show that for every finite field extension L of K, we have $(L^o)^\flat = \overline{\mathbb{F}}_p$.

Problem 15

Let K be a field, and let A be a subring of K. Show that the following conditions are equivalent:

(i) For every $x \in K$, $x \in A$ or $x^{-1} \in A$.

(ii) There is an absolute value $|\cdot|$ on K, with values in $\Gamma \sqcup \{0\}$ for some totally ordered group Γ (cf. Pbm. 12), whose valuation ring $\{x \in K;\ |x| \leqslant 1\}$ is A.

Hint: To prove that (i)$\Rightarrow$(ii), let $\Gamma = K^\times/A^\times$ with ordering $\overline{x} \leqslant \overline{y} :\Leftrightarrow xy^{-1} \in A$.

Problem 16

Let K be a field, and let A be a subring of K which satisfies the equivalent conditions of Problem 15. Assume that A is noetherian. Show that A is a local principal domain (and hence, a discrete valuation ring).