Perfectoid spaces PD Dr. A. Chatzistamatiou
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Problem sheet 8

Due date: Dec. 8, 2017

Problem 29

Show that every étale morphism of rings in characteristic p is relatively perfect. Construct a flat R-algebra that is relatively perfect, but not étale.

Problem 30

Find all flat deformations of $R := \mathbb{F}_p[x_1, \dots, x_n]$ to $\mathbb{Z}/p^2$ and compute their automorphism groups. Find all possible lifts of the Frobenius morphism.

Do the same for $R := \mathbb{F}_p[x_1^{\frac{1}{p^\infty}}, \dots, x_n^{\frac{1}{p^\infty}}]$.

Problem 31

Recall the specialization order on a topological space X: For $x, y \in X$, we say that y is a specialization of x if $y \in \overline{\{x\}}$, i.e., y lies in the closure of $\{x\}$. If X is T_0, then this defines a partial order on the set X.

Now let X be a partially ordered set. Show that there is at most one way to equip X with a topology such that the topological space X is spectral and noetherian and that the specialization order is the given partial order on X.

Problem 32

Prove, without invoking Hochster's theorem, that there exists a ring A such that $\operatorname{Spec}(A)$ is homeomorphic to $\operatorname{Spv} \mathbb{Z}$.