Perfectoid spaces PD Dr. A. Chatzistamatiou
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Problem sheet 9

Due date: Dec. 15, 2017

Problem 33

Let K be a perfectoid field. Show that if $\mathrm{char}(K) = p > 0$ then a K-Banach algebra is perfectoid if and only if it is uniform and perfect.

Problem 34

Let K be a perfectoid field, and let π be a pseudouniformizer. For an étale $K^o[x_1, \dots, x_n]$-algebra A', we set

$$A := \pi\text{-adic completion of } K^o[x_1^{\frac{1}{p^\infty}}, \dots, x_n^{\frac{1}{p^\infty}}] \otimes_{K^o[x_1,\dots,x_n]} A'$$

Show that $K \otimes_{K^o} A$ (equipped with the norm from Exercise (4.5)) is perfectoid.

Problem 35

Let A be a ring, let $v \in \mathrm{Spv}(A)$, and let $H \subseteq \Gamma_v$ be a convex subgroup with $c\Gamma_v \subseteq H$. Define

$$v_{|H} \colon A \to H \cup \{0\}, \quad a \mapsto \begin{cases} v(a) & \text{if } v(a) \in H \cup \{0\}, \\ 0 & \text{if } v(a) \notin H. \end{cases}$$

Then $v_{|H}$ is an element of $\mathrm{Spv}(A)$ (you do not have to show this).

Prove that $v_{|H}$ is a specialization of v, i.e., $v_{|H} \in \overline{\{v\}}$.

Problem 36

Let K be a field which is equipped with a complete rank 1 non-archimedean absolute value. Let

$$C = \left\{ \sum_{i \geqslant 0} a_i T^i \in K^\circ\langle T\rangle;\ \forall i \geqslant 1 : |a_i| < 1 \right\},$$

a subring of $K^\circ\langle T\rangle$. Show that $(K\langle T\rangle, K^\circ\langle T\rangle)$ and $(K\langle T\rangle, C)$ are Huber pairs and that $\mathrm{Spa}(K\langle T\rangle, K^\circ\langle T\rangle) \neq \mathrm{Spa}(K\langle T\rangle, C)$. (*Hint:* Look for a suitable modification of the construction in Problem 12.)