Blockseminar Düsseldorf
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HYPERBOLICITY AND JET DIFFERENTIALS

Abstract. We would like to learn about hyperbolicity and jet differentials using Demailly's lecture notes [1]. Our concrete goal is to understand the proof of the Bloch Theorem, which says that the (analytic) Zariski closure of an entire curve on a complex torus is a translate of a subtorus.

Our main reference are the lecture notes [1] by Demailly.

Talk 1 (Hyperbolicity and the Ahlfors-Schwarz Lemma — 60 min)**.** (Christian Liedtke) Introduce the concept of directed manifolds (X, V) and Brody hyperbolicity (§1) and prove the Ahlfors-Schwarz Lemma (3.2), recalling/introducing the analysis needed (plurisubharmonic functions, curvature...).

Talk 2 (Jet bundles — 45 min)**.** (Jan Christian Rohde) Define the k-jet bundles J_k and the projectivized k-jet bundles P_kV associated to a directed manifold (X, V) and describe the fibers $\mathbb{R}_{r,k}$ (Page 28, line 8). Mention the exact sequences (5.4) and (5.4') and define $f_{[k]} : \Delta_R \to P_kV$ and $f'_{[k-1]}$ induced from a tangent trajectory $f : \Delta_R \to X$. (First three pages of §5, you will also need some material from pages 20–21.)

Talk 3 (Jet differentials — 60 min)**.** (Franziska Heinloth) Prove the relative ampleness of certain natural line bundles on the projectivized jet bundles P_kV (Prop. 6.16, iii)). Give an interpretation of the push forward of $\mathcal{O}_{P_kV}(m)$ in terms of invariant jet differentials and describe its relative base locus (Theorem 6.8, ii) and iii)).

Talk 4 (Jet metrics — 60 min)**.** (Martin Möller) Construct a singular metric with positive curvature and given degeneration set as in Prop. 7.2, ii). Then prove the following central result: If $\mathcal{O}_{P_kV}(-1)$ carries a hermitian metric with negative curvature, then for every entire curve $f : \mathbb{C} \to X$ tangent to V the image of the induced morphism $f_{[k]} : \mathbb{C} \to P_kV$ is contained in the singularity set of the metric.

Talk 5 (The Bloch Theorem — 45 min)**.** (Eckart Viehweg) Prove the Bloch Theorem 9.1. If time permits, give also the corollaries, and/or an overview of other hyperbolicity concepts, their relation, conjectures of Green–Griffiths and Lang ...

References

[1] Jean-Pierre Demailly. Algebraic criteria for Kobayashi hyperbolic projective varieties and jet differentials. In *Algebraic geometry—Santa Cruz 1995*, volume 62 of *Proc. Sympos. Pure Math.*, pages 285–360. Amer. Math. Soc., Providence, RI, 1997. available at `http://www-fourier.ujf-grenoble.fr/~demailly/articles.html`

[2] Claire Voisin. *On some problems of Kobayashi and Lang; algebraic approaches.* In *Current developments in mathematics, 2003*, pages 53–125. Cambridge University Press, Cambridge, 2003. Int. Press, Somerville, MA, 2003. available at `http://www.math.jussieu.fr/~voisin/listepublications.html`