

# Boundedness conditions for semistable sheaves in all characteristics

FORSCHUNGSSEMINAR DER ARBEITSGRUPPE ALGEBRAISCHE GEOMETRIE<sup>1 2 3 4</sup>  
UNIVERSITÄT DUISBURG–ESSEN, FALL 2004  
TUESDAYS 2:15—3:45PM, LOCATION: T03 R03 D56

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Our goal is to work through the article of Langer [Lan04] where he shows *boundedness* for semistable sheaves in all characteristics. To understand this properly we first spend some time with semistable sheaves and their moduli spaces, roughly following Chapters 1–4 of the book [HL97].

The dates of the talks are tentative as things will most likely shift (for example Dec 23 is most likely skipped for sentimental reasons). If we happen to have extra time towards the end of the term there is ample supply of additional related topics one could take up.

- Oct 12 (**Alexander Schmitt**) Introduction to this semesters seminar.
- Oct 19 & 26 (**Wioletta Syzdek and Stefan Kukulies**) Sections 1.2–1.4 of [HL97] and the Harder–Narasimhan polytope from [Sha77]: Basics of stability (focusing on the torsion free case). Gieseker and  $\mu$ –(semi)stability. Harder–Narasimhan filtration. Harder–Narasimhan polytope, in particular [Sha77, Theorem 2]. Example: The cotangent bundle on  $\mathbb{P}^n$  is stable.
- Nov 2 (**Hendrik Russel**) Boundedness [HL97] Section 1.7 and parts of Section 3.3 (Lemma 3.3.2). The answer to: What is meant by boundedness? Castelnuovo–Mumford regularity, Kleiman Criterion. . . . From [HL97, Section 3.3] Theorem 3.2.1 with indication of proof.
- Nov 9 & 16 (**Kai Ruelling and Bjoern Buth**) Quot-Scheme. Sections 2.1–2.2 of [HL97]. Flat families of coherent sheaves, Grassmanian varieties and the Quot scheme. This is the fundamental tool for the construction of the moduli spaces later and the talk might be somewhat longer than the proposed 90 minutes.
- Nov 23 (**Martin Moeller**) Bogomolov inequality. [HL97] Section 3.4. Tensor products of stable sheaves. Numerical consequence of semistability (Bogomolov).
- Nov 30 (**Ho Hai Phung**) Moduli spaces – construction. Section 4 in [HL97].

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<sup>1</sup>as of January 12, 2005, the members are: **Esnault, Viehweg**, Bittner, Blickle, Chatzistamatiou, Kuronya, Kukulies, Lebelt, Möller, Phung, Rülling, Russell, Schmitt, Syzdek, Szemberg

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<sup>3</sup>Organizers for WS04/05 are Manuel Blickle (bm0056) and Alexander Schmitt (mat907).

<sup>4</sup>Further participants came from the Institut für experimentelle Mathematik and Universität Düsseldorf

- Nov 7 (**Thomas Szemberg**) Moduli spaces – examples.
- Dec 14 (**Manuel Blickle**) Semistability and Frobenius as in Section 2 of [Lan04]. Cartier descent. Strong Harder–Narasimhan filtration and consequences.
- Jan 11 **Oberseminar 5–6pm: Holger Brenner** will speak on an application of Langer’s results to Hilbert–Kunz functions.
- Jan 11 & 18 (**Franziska Bittner and Alex Küronya**) Bogomolov’s inequality and restriction in all characteristics. [Lan04], Section 3. This is the technical heart of Langer’s paper where he shows the mentioned results independent of the characteristic. Best tackled by two (or more) people.
- Jan 18 (**Adrian Langer**) Consequences of his boundedness result.

#### REFERENCES

- [Fri98] Robert Friedman, *Algebraic surfaces and holomorphic vector bundles*, Universitext, Springer-Verlag, New York, 1998.
- [HL97] Daniel Huybrechts and Manfred Lehn, *The geometry of moduli spaces of sheaves*, Aspects of Mathematics, E31, Friedr. Vieweg & Sohn, Braunschweig, 1997, available at [www.mathematik.uni-mainz.de/~lehn/Arbeiten/shaves.ps](http://www.mathematik.uni-mainz.de/~lehn/Arbeiten/shaves.ps).
- [Lan04] Adrian Langer, *Semistable sheaves in positive characteristic*, *Ann. of Math. (2)* **159** (2004), no. 1, 251–276.
- [LP97] J. Le Potier, *Lectures on vector bundles*, Cambridge Studies in Advanced Mathematics, vol. 54, Cambridge University Press, Cambridge, 1997, Translated by A. Maciocia.
- [OSS80] Christian Okonek, Michael Schneider, and Heinz Spindler, *Vector bundles on complex projective spaces*, Progress in Mathematics, vol. 3, Birkhäuser Boston, Mass., 1980.
- [Sha77] Stephen S. Shatz, *The decomposition and specialization of algebraic families of vector bundles*, *Compositio Math.* **35** (1977), no. 2, 163–187.