Title: Meromorphic differentials with twisted coefficients on compact Riemann surfaces

Abstract: In this talk, I'll consider the construction of meromorphic differentials with twisted coefficients on compact Riemann surfaces $\overline{X}$ by using classical Dirichlet's Principle.

Our starting point is a semi-simple representation ρ into the general linear groups $GL(n, \mathbb{C})$ of $\pi_1(\overline{X})$, equivalently, a local system L_ρ over $\overline{X}$, the induced representation of which is ρ. Then, our *twisted coefficient* is $End(L_\rho)$. A metric h on L_ρ can be canonically explained as a ρ-equivariant map, still denoted by h, from the universal covering of $\overline{X}$ into $GL(n, \mathbb{C})/U(n)$, the differential $(\partial h)h^{-1}$ of which is a $(1,0)$-form valued in $End(L_\rho)$. If, furthermore, h is a harmonic map (metric), the differential $(\partial h)h^{-1}$ is holomorphic. If one considers harmonic metrics with certain singularities on some points of $\overline{X}$, one will get meromorphic differentials with singularities on those points, the residues of which have zero or non-zero *real* eigenvalues; this generalizes the classical theory of meromorphic differential on compact Riemann surfaces. We will mainly discuss meromorphic differentials with regular singularities; we'll see these singularities corresponds to *regular filtered structures* on the local system defined by C. Simpson.

I'll also discuss more general cases. Choose arbitrarily some points $p_1, \cdots, p_s$ in $\overline{X}$ and assume a semi-simple representation ρ from $\pi_1(\overline{X} \setminus \{p_1, \cdots, p_s\})$ into $GL(n, \mathbb{C})$. One has two interesting special cases to consider: the images of the representation restricted to punctured disks at all p_i are
1) hyperbolic; 2) unipotent.
(Actually, one also should consider the case of elliptic images, but it can be easily covered in the argument of representations of $\pi_1(\overline{X})$). We will mainly discuss the case 1) (since the case 2) is easy and in principle the same as the period mapping theory); we'll see that if we assume trivial growth order on the metric (trivial filtered structure; a typical model is the radial projection of the punctured disk to the circle), one will obtain meromorphic differentials, the residues of which have *pure imaginary* eigenvalues. This is essentially due to Jost-Zuo (1997).