Étale cohomology for rigid analytic spaces

FORSCHUNGSSEMINAR DER ARBEITSGRUPPE ALGEBRAISCHE GEOMETRIE[1 2 3 4]
UNIVERSITÄT DUISBURG–ESSEN
TUESDAY 2:15—3:45PM, LOCATION: T03 R03 D56

The goal of the seminar is to understand the content of Berkovich' article [2]. In it Berkovich develops étale cohomology and proves the basic theorems which are classical for schemes: constructibility of higher direct images of sheaves, Poincaré Duality, invariance under algebraically closed base change, smooth and proper base change and a comparison theorem. Before reading this paper, we will have a concise introduction to Berkovich' theory of rigid analytic spaces, following [1]. We concentrate on examples. A short overview of the theory is given in [3]. It may be helpful for the participants to read this paper before the seminar, since the many definitions take some time to get used to. As standing assumptions, we suppose that k is a non-archimedean field with non-trivial valuation. Moreover, all k-analytic spaces are supposed to be good in the sense of [2]. In the unlikely event that time remains at the end, we would like to discuss vanishing cycles.

April 19 (**Rohde**) The spectrum of a Banach ring: definition. The reference for this talk is [1, Chapter 1]. Explain the idea of Theorem 1.2.1 and Remark 1.2.2.i. Important are Corollary 1.2.4, Remark 1.2.5, Theorem 1.3.1. Example 1.4.4 should be discussed thoroughly, but just for k a non-archimedean field. If time allows, one could discuss Definition 1.5.1 and the remark that follows.

April 26 (**Buth**) In this talk the basic definitions of affinoid algebra, affinoid domain and k-analytic space should be explained. The challenge is to focus on the concepts, and leave the technical details for later talks. Discuss the definitions as given in [3, Section 2], and refer for details to [1, Chapter 2 and Section 3.1].

The definition of k-analytic space of [3] coincides with the one given in [2] which is more general than the one of [1] (see [2, Section 1.5]). In this seminar we suppose that all k-analytic spaces are good [2, 1.2.16]); this coincides with the definition of k-analytic space in [1]. Mention [2, Prop. 1.3.4].

Discuss Properties (1)–(5) of [3, Section 2].

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[1]as of May 25, 2005, the members are: **Esnault, Viehweg**, Blickle, Chatzistamatiou, Heinloth, Heinloth, Kuronya, Kukulies, Lebelt, Möller, Phung, Rhode, Rülling, Russell, Schmitt, Syzdek, Szemberg

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[3]Organizers for SS05 are Irene Bouw (bouw@math.uni-duesseldorf.de) and Stefan Wewers (wewers@math.uni-bonn.de).

[4]Further participants came from the Institut für experimentelle Mathematik and Universität Düsseldorf

May 3 (**Diem**) The Tate elliptic curve $\mathcal{T}$. We suggest that the speaker defines $\mathcal{T}$ as $\mathbb{G}_{m,k}^{\mathrm{an}}/\ll q \gg$, following [4, Section 5.1]. Define the covering by ring domains as in [4, p. 122], but leave out details!

Show that $\mathcal{T}$ is homotopy equivalent to a circle ([3, Section 2, Property 7]). To show this, state the fact that if X is a smooth analytic space, then the underlying topological space $|X|$ is locally contractible ([1, Section 4.2]). The topological space $|X|$ is homotopy equivalent to the dual graph of the semistable reduction of X ([3, Section 5]). Here one uses the statement of the semistable reduction theorem for curves. State all used theorems without proof, and try to avoid technical details and complicated notation.

Also discuss Property (8) of [3, Section 2].

May 10 (**Chatzistamatiou**) Description of rigid analytic spaces via formal schemes. Discuss the content of [5]. State the comparison result, due to Bosch–Lütkebohmert [3, Section 2, Property 10]. Also discuss Property 9 and the remark of [3, Section 2]. Describe the formal model of the Tate elliptic curve.

May 24 (**Liedtke**) Describe the reduction and formal model of $\mathbb{P}^1$ minus a set $\mathcal{L}$ of k-rational points ([4, Section 4.9]). A key example is the Drinfeld upper half plane which corresponds to $\mathcal{L}$ equal to the set of all k-rational points. Give the ad hoc definition of the first étale cohomology group due to Drinfeld ([4, Sections 2.7 and 8.4]), and the description of the cocains.

May 31 (**Russel**) Étale morphisms. Reference: Sections 3.1–3.4 and 2.1–2.3 of [2]. Everything that is similar to the corresponding statements for schemes should be kept short.

(a) Define finite and quasi-finite morphisms (Def. 3.1.1). Summarize the following results: Cor. 3.1.3, Prop. 3.1.4, Cor. 3.1.6, Cor. 3.1.10.

(b) Define étale morphisms (Def. 3.3.4), and summarize Cor. 3.3.6, Cor. 3.3.8, Cor. 3.3.9, Prop. 3.3.10, Prop. 3.3.11.

(c) Explain the proof of Theorem 3.4.1. It relies on the results of Sections 2.1–2.3 which should be explained as necessary.

June 7 (**Rülling**) Étale cohomology. Reference: Sections 4.1 and 4.2 of [2]. Everything that is similar to the corresponding statements for schemes should be kept short.

(a) Introduce étale topology, étale site, étale topos, $H^q(X, F)$. Cor. 4.1.2, Prop. 4.1.3. (Note: in talk 2 we have seen that $X_G = X$ since we assume that X is good.)

(b) Example 4.1.6, Prop. 4.1.7 Prop. 4.1.10, Cor. 4.3.8 (for good k-analytic spaces this is already explained after Cor. 4.1.11). This is the central part of the talk. It should be done as detailed as time allows.

(c) The definition of the stalk of sheaf (Section 4.2). Only consider the case of germs (X, x). Mention Prop. 4.2.1 and Cor. 4.2.3. Prop. 4.2.4, Theorem 4.2.6 and Theorem 4.2.7 are important for us and should be well-understood.

June 14 Arbeitstagung.

June 21 Extra Oberseminar.

June 28, 14:00 (**Möller**) The comparison theorem ([2, Theorem 7.1.1]).

(a) Give a short explanation of cohomology with support, avoiding technicalities ([2, Sections 5.1–5.3]). Only discuss the following situation. Let X be a compact k-analytic space, for example $X = \mathcal{X}^{\mathrm{an}}$ for a projective scheme $\mathcal{X}$, or X an affinoid. Let $Z \subset X$ be a closed analytic subset and $U = X - Z$ the complement. Denote by $j : U \hookrightarrow X$ and $i : Z \hookrightarrow X$ the inclusions. Let F be an abelian sheaf on X. As for schemes, the short exact sequence $0 \to j_! F \to F \to i_* F \to 0$ induces a long exact sequence $H^q_c(U, F) \to H^q(X, F) \to H^q(Z, i^* i_* F)$. This is a special case of Prop. 5.2.6.ii.

(b) Explain the proof of the comparison theorem for curves (Theorem 6.1.1)

(c) Give an overview of the proof of the comparison theorem in the general case (Theorem 7.1.1).

June 28, 17:00 (**Kuronya**) Étale cohomology of k-analytic curves I.

(a) Give a short review of standard and elementary curves ([2, Section 3.6]; things simplify since we assume that the valuation on k is nontrivial, and the k-analytic space is good.)

(b) Introduce the trace map. Discuss Theorem 6.2.1 of [2] as detailed as time allows. This is a key ingredient for the Poincaré duality which we discuss in the last talk.

July 5 (**F. Heinloth**) Étale cohomology of k-analytic curves II. This talks gives analogs of classical theorems for curves, like Riemann's Existence Theorem. Discuss Theorems 6.3.2, 6.3.9, 6.4.1 and Remark 6.4.2 of [2].

July 12 (**J. Heinloth/Wewers**) The main theorems. The goal is to explain a selection of Sections 7.2–7.8 of [2]. Ideally, the speaker should have a solid knowledge of étale cohomology for schemes, as in SGA IV.

References

[1] V. G. Berkovich. *Spectral theory and analytic geometry over non-Archimedean fields*, volume 33 of *Mathematical Surveys and Monographs*. American Mathematical Society, 1990.

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[4] J. Fresnel and M. van der Put. *Rigid analytic geometry and its applications*, volume 218 of *Progress in Mathematics*. Birkhäuser Boston Inc., 2004.

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