Universität Duisburg-Essen WS 2006/07

Arbeitsgemeinschaft: Rational curves

Organisation: M. Blickle, J. Heinloth, M. Möller, E. Viehweg

Do 14.15-15.00, 15.15-16.00 , T03 R03 D26

Overview

This winter's program consists of several individual parts, all of which are dominated by rational curves.

The first part (talks 1–6) aims to show the existence of a section of a fibration $f : X \to Y$ whose general fibre is rationally connected. As a corollary we obtain that if moreover Y is rationally connected, then X is rationally connected, too. This corollary allows to define the rational quotient $R(X)$ of X, usually characterized by a universal property involving fibrations with rationally connected general fibre, as the fibration $r_X : X \to R(X)$ such that r_X has rationally connected general fibre and such that $R(X)$ is not uniruled. We discuss the construction of the rational quotient as the starting point to Campana's core (talk 9). Talks 1 and 2 introduce general techniques related to rational curves. In talks 3–4 we follow the geometric approach of Graber, Harris and Starr ([GHS]) valid over the complex numbers. In talks 5-6 we generalize the same statement to an arbitrary base field, following the less geometric arguments in [dJS].

Instead of just one section weak approximation asks for the existence of sections passing through prescribed points and moreover with prescribed tangency conditions. In second part (talks 7–8) we discuss the results of Hassett and Tschinkel (extending work of Kollár, Miyaoka, Mori) showing that weak approximation holds, imposing some restrictions if the prescribed points lie on singular fibres. If someone with number theory background is willing to give a talk, it might be interesting to compare the function field situation with the number field situation (Skinner's circle method). Otherwise we skip talk 8.

The third part (talks 9–12) aims to construct the core of a variety in the sense of Campana. Campana's idea is to use the rational quotient and the Iitaka-fibration for classification theory as follows. For any projective variety X there is a sequence of fibrations $X \to X_1 \to \cdots \to X_k =: C(X)$ such that the general fibre of each fibration are of Kodaira-dimension zero or rationally connected varieties and such that the core $C(X)$ accounts for the 'part' of general type in X. In order to obtain a useful notion of core, e.g. such that its dimension is stable under étale coverings of X, Campana is lead to introduce orbifold structures on the bases of fibrations.

Of course classification theory of manifolds has to adress classification of the building blocks (Kodaira-dimension zero, rationally connected varieties and general type) but this is not the aim of Campana's paper.

In a paper by S. Lu ([Lu]) ideas in the same spirit are developed and some of Campanas (many) conjectures are solved. Parallel reading of the paper might be useful.

19.10.06 **1. Rational connectedness**

Define (seperable) rational connectedness, rational chain connectedness unir-uled varieties. Give an overview over the implications that do and do not hold in characteristic zero (positive characteristic issues will be dealt with in talk 5 and 6).

Define (very) free curves. Prove [De2] Prop. 4.9 and Cor. 4.17.

Ref.: [De2], §4, [De1], [De3], [Ko]

Henrik Rusell

26.10.06 **2. Overview over implications in characteristic zero, examples**

Statement of the main smoothing result, to be shown in talk 4 ([De2] Cor. 4.28 or [De3] Thm. 2.2). With the help of this result all variations of rational connectedness and rational chain connectedness (using families, connecting generic points, connecting any pair of points) will be seen to be equivalent for smooth varieties over an uncountable field of characteristic zero

The locus X^{free} plays a key role and explains the restriction to characteristic zero.

Examples for the structure of X^{free}. Example: A Fano variety.

Ref.: [De2] Martin Möller

02.11.06 **3. Producing sections I (over $\mathbb{C}$)**

This is logically the 4th talk.

State the main result of [GHS], mention some consequences. Reduction to a rational base.

Construct a section from a convenient (conditions ($C1$) and ($C2$) in [De3]) multisection admitting the existence of moduli space of stable maps. Construct a convient multisection by smoothing a comb.

Ref.: [De3] §4.1 and §4.2 , [GHS]

Dmitry Doryn

09.11.06 **4. Smoothability, chain connectedness in characteristic zero**

This is logically the 3th talk.

Prove smoothability of rational trees and combs. The aim is a) the equivalence of rational connectedness and chain connectedness in characteristic zero and b) on a rationally connected variety the existence of a rational curve passing through given points with tangency conditions ([De3] Thm. 2.2).

Ref.: [De2], §4.4, [Ko]

Jan Christian Rohde

16.11.06 **5. Producing sections II (over $\mathbb{C}$)**

Create more ramification points and use them to deal with multiple fibres.
Ref.: [De3] §4.3 and §4.4, [GHS]

Franziska Heinloth

23.11.06 **6. Producing sections III (over general base fields)**

The summary of [dJS] in [De3] §5 seems a lot for one talk but hardly enough

for two talks. Add counterexamples for the implications in talk 1, that are no longer valid in positive characteristic (e.g. [De2] §4.4 and what else?). It is up to the speaker or pair of speakers to decide how long he/she/they need.
Ref.: [De3] §5 [dJS]

Manuel Blickle

30.11.06 **7. Producing sections IV (over general base fields)**
See comment to talk 6.

Ref.: [De3] §5 [dJS]

Jochen Heinloth

07.11.06 **8. Weak approximation over function fields**
Give an overview over the results [HT1], [HT2] and [HT3] illustrating the main techniques.
Ref.: [HT1], [HT2] and [HT3], [KMM]

Stefan Kukulies

14.12.06 **9. Weak approximation over number fields**
Explain the techniques available over number fields. Discuss with speaker of talk 7 who gives a comparison with [HT3].
Some of us are absent this date. Probably this lecture is moved to Mo/Tue 18./19.12.06 or to 11.01.07.

Ref.: [Sk]

Pedro L. del Angel

11.01.07 **10. The rational quotient**
Just construct it!
Ref.: [De2] §5, [KMM], [Ca1]

Alexander Schmitt

18.01.07 **11. Orbifold structures for fibrations: Why and how?**
Give the example why the natural idea using rational quotient and Iitaka fibration does not produce the desired core. Introduce orbifold structures and the Kodaira-dimension of a fibration. (Sounds not enough for one talk yet. Maybe include a discussion of κ-RC, RC and conjecture $-\infty$, see [Ca2] 1.10 and §2.A)
Ref.: [Ca3] §2.C [Ca2], [Lu]

N.N.

25.01.07 **12. Orbifold version of Iitakas conjecture**
Ref.: [Ca2]

Eckart Viehweg

08.02.07 **13. Special varieties and Campana's core**
This heavily depends on the number of speakers willing to adress the topic.
Ref.: [Ca3], [Ca2], [Lu]

N.N.

15.02.07 **14. ??**
Ref.: ??

N.N.

References

[Ca1] Campana, F., *Coréduction algébrique d'un espace analytique faiblement Kählérien compact*, Invent. Math. 63 (1981), 187–223

[Ca2] Campana, F., *Special varieties and classification theory*, math.AG/0110051, Ann. Sci. Fourier 54 (2004), 499–655

[Ca3] Campana, F., *Orbifolds, variétés speciales et classification des variétés kählerieness compactes*, preprint math.AG/0402242 (2004)

[De1] Debarre, O., *Variétés de Fano*, Exp. 827, Sém. Bourbaki 1996/97, Astérisque 245 (1997), 197–221

[De2] Debarre, O., *Higher dimensional algebraic geometry*, Universitext, Springer (2001)

[De3] Debarre, O., *Variétés rationellement connexes*, Exp. 905, Sém. Bourbaki 2001/02, Astérisque 290 (2003), 243–266

[dJS] de Jong, A.J., Starr, J., *Every rationally connected variety over the function field of a curve has a rational point*, Amer. J. Math. 125 (2003), 567–580

[GHS] Graber, J. Harris, J. Starr, *Families of rationally connected varieties*, math.AG/0109220, J. Amer. Math. Soc. 16 (2003), 57–67

[HT1] Hassett, B., Tschinkel, Y., *Weak approximation over function fields*, Invent. Math. 163 (2006), 171–190

[HT2] Hassett, B., Tschinkel, Y., *Approximation at places of bad reduction for rationally connected varieties*, preprint (2005)

[HT3] Hassett, B., Tschinkel, Y., *Weak approximation for hypersurfaces of low degree*, preprint (2006)

[KMM] Kollár, J., Miyaoka, Y., Mori, S., *Rationally connected varieties*, J. Alg. Geom. 1 (1992), 429–448

[Ko] Kollár, J., *Rational curves on algebraic varieties*, Erg. der Math. 32, Springer (1996)

[Lu] Lu, S.S. *A refined Kodaira dimension and its canonical fibration*, preprint math.AG/0211029 (2002)

[Sk] Skinner, C.M., *Forms over number fields and weak approximation*, Comp. Math. 106 (1997), 11–29

Debarre's Bourbaki articles are available on his web site.